MONOTONIC INCREASING AND DECREASING SEQUENCES OF S(n)

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Problem (11)

Monotonic increasing and monotonic decreasing sequences of S(n) were investigated for x >= 6.

First number (i) in range	number of sequences S x = 6 inc	x = 6 dec	x = 7 inc	x = 7 dec	x = 8 inc	x = 8 dec	x = 9 inc	x = 9 dec
1 – 499 999	75	83	7	10	0	2	0	0
500 000 – 999 999	80	76	14	18	1	3	1	1
1 000 000 – 1 499 999	75	63	8	10	1	2	1	1

There appears to be no evidence for a largest value for x. The sequences for x = 9 are shown in Results Table 1.

The existence of sequences with the same first order finite differences was then considered eg:

i = 440	S(i) = 11;	i = 5073	S(i) = 89
i+1 = 441	S(i+1) = 14;	i+1 = 5074	S(i+1) = 59
i+2 = 442	S(i+2) = 17;	i+2 = 5075	S(i+2) = 29

Apart from the initial quartet 2,3,4,5 all such sequences with i < 1 000 000 are triplets. If the first order finite differences are multiples of 6 then the S(n) values appear to be prime numbers. The values are shown in Results Table 2.

RESULTS TABLE 1

Sequences of S(n)

x = 9

n = 586951 S(n) = 586951
n = 586952 S(n) = 73369
n = 586953 S(n) = 21739
n = 586954 S(n) = 9467
n = 586955 S(n) = 1319
n = 586956 S(n) = 1193
n = 586957 S(n) = 1181
n = 586958 S(n) = 1091
n = 586959 S(n) = 677
n = 586960 S(n) = 29

x = 9

n = 721970 S(n) = 73
n = 721971 S(n) = 827
n = 721972 S(n) = 907
n = 721973 S(n) = 6067
n = 721974 S(n) = 10939
n = 721975 S(n) = 28879
n = 721976 S(n) = 90247
n = 721977 S(n) = 240659
n = 721978 S(n) = 360989
n = 721979 S(n) = 721979

x = 9

n = 1091150 S(n) = 157
n = 1091151 S(n) = 709
n = 1091152 S(n) = 1451
n = 1091153 S(n) = 1607
n = 1091154 S(n) = 6271
n = 1091155 S(n) = 16787
n = 1091156 S(n) = 24799
n = 1091157 S(n) = 363719
n = 1091158 S(n) = 545579
n = 1091159 S(n) = 1091159

x = 9

n = 1473257 S(n) = 1473257
n = 1473258 S(n) = 8467
n = 1473259 S(n) = 6323
n = 1473260 S(n) = 3877
n = 1473261 S(n) = 3533
n = 1473262 S(n) = 2239
n = 1473263 S(n) = 1999
n = 1473264 S(n) = 787
n = 1473265 S(n) = 557
n = 1473266 S(n) = 463

RESULTS TABLE 2

same difference = 1 n = 4 S(n-2) = 2 S(n-1) = 3 S(n) = 4

same difference = 1 n = 5 S(n-2) = 3 S(n-1) = 4 S(n) = 5

same difference = 11 n = 18 S(n-2) = 6 S(n-1) = 17 S(n) = 6

same difference = 3 n = 442 S(n-2) = 11 S(n-1) = 14 S(n) = 17

same difference = 30 n = 5075 S(n-2) = 89 S(n-1) = 59 S(n) = 29

same difference = 60 n = 6409 S(n-2) = 149 S(n-1) = 89 S(n) = 29

same difference = 48 n = 6479 S(n-2) = 127 S(n-1) = 79 S(n) = 31

same difference = 36 n = 8177 S(n-2) = 109 S(n-1) = 73 S(n) = 37

same difference = 84 n = 13717 S(n-2) = 211 S(n-1) = 127 S(n) = 43

same difference = 168 n = 20468 S(n-2) = 379 S(n-1) = 211 S(n) = 43

same difference = 210 n = 22591 S(n-2) = 461 S(n-1) = 251 S(n) = 41

same difference = 120 n = 35145 S(n-2) = 311 S(n-1) = 191 S(n) = 71

same difference = 180 n = 59719 S(n-2) = 449 S(n-1) = 269 S(n) = 89

same difference = 150 n = 67771 S(n-2) = 401 S(n-1) = 251 S(n) = 101

same difference = 264 n = 73425 S(n-2) = 617 S(n-1) = 353 S(n) = 89

same difference = 840 n = 74005 S(n-2) = 1721 S(n-1) = 881 S(n) = 41

same difference = 24 n = 82297 S(n-2) = 151 S(n-1) = 127 S(n) = 103

same difference = 60 n = 104669 S(n-2) = 251 S(n-1) = 191 S(n) = 131

same difference = 330 n = 111507 S(n-2) = 769 S(n-1) = 439 S(n) = 109

same difference = 36 n = 114427 S(n-2) = 199 S(n-1) = 163 S(n) = 127

same difference = 252 n = 120523 S(n-2) = 631 S(n-1) = 379 S(n) = 127

same difference = 120 n = 129928 S(n-2) = 389 S(n-1) = 269 S(n) = 149

same difference = 952 n = 146004 S(n-2) = 1973 S(n-1) = 1021 S(n) = 69

same difference = 600 n = 153520 S(n-2) = 1301 S(n-1) = 701 S(n) = 101

same difference = 12 n = 180482 S(n-2) = 47 S(n-1) = 59 S(n) = 71

same difference = 660 n = 181485 S(n-2) = 1429 S(n-1) = 769 S(n) = 109

same difference = 60 n = 189954 S(n-2) = 53 S(n-1) = 113 S(n) = 173

same difference = 90 n = 192067 S(n-2) = 359 S(n-1) = 269 S(n) = 179

same difference = 324 n = 198697 S(n-2) = 811 S(n-1) = 487 S(n) = 163

same difference = 336 n = 209752 S(n-2) = 839 S(n-1) = 503 S(n) = 167

same difference = 228 n = 227099 S(n-2) = 647 S(n-1) = 419 S(n) = 191

same difference = 150 n = 231039 S(n-2) = 499 S(n-1) = 349 S(n) = 199

same difference = 264 n = 253725 S(n-2) = 727 S(n-1) = 463 S(n) = 199

same difference = 210 n = 266915 S(n-2) = 631 S(n-1) = 421 S(n) = 211

same difference = 648 n = 297638 S(n-2) = 1459 S(n-1) = 811 S(n) = 163

same difference = 2808 n = 306128 S(n-2) = 5669 S(n-1) = 2861 S(n) = 53

same difference = 1320 n = 324384 S(n-2) = 2749 S(n-1) = 1429 S(n) = 109

same difference = 18 n = 326163 S(n-2) = 199 S(n-1) = 181 S(n) = 163

same difference = 240 n = 342965 S(n-2) = 719 S(n-1) = 479 S(n) = 239

same difference = 36 n = 346390 S(n-2) = 139 S(n-1) = 103 S(n) = 67

same difference = 300 n = 386906 S(n-2) = 47 S(n-1) = 347 S(n) = 647

same difference = 840 n = 409422 S(n-2) = 1861 S(n-1) = 1021 S(n) = 181

same difference = 270 n = 440375 S(n-2) = 811 S(n-1) = 541 S(n) = 271

same difference = 936 n = 443450 S(n-2) = 2053 S(n-1) = 1117 S(n) = 181

same difference = 120 n = 443850 S(n-2) = 509 S(n-1) = 389 S(n) = 269

same difference = 792 n = 443969 S(n-2) = 1783 S(n-1) = 991 S(n) = 199

same difference = 450 n = 450043 S(n-2) = 1151 S(n-1) = 701 S(n) = 251

same difference = 306 n = 451215 S(n-2) = 883 S(n-1) = 577 S(n) = 271

same difference = 210 n = 460559 S(n-2) = 701 S(n-1) = 491 S(n) = 281

same difference = 240 n = 464212 S(n-2) = 761 S(n-1) = 521 S(n) = 281

same difference = 360 n = 470727 S(n-2) = 991 S(n-1) = 631 S(n) = 271

same difference = 624 n = 473922 S(n-2) = 1481 S(n-1) = 857 S(n) = 233

same difference = 90 n = 481779 S(n-2) = 449 S(n-1) = 359 S(n) = 269

same difference = 126 n = 511688 S(n-2) = 131 S(n-1) = 257 S(n) = 383

same difference = 672 n = 512894 S(n-2) = 1583 S(n-1) = 911 S(n) = 239

same difference = 480 n = 521946 S(n-2) = 1231 S(n-1) = 751 S(n) = 271

same difference = 714 n = 531775 S(n-2) = 1667 S(n-1) = 953 S(n) = 239

same difference = 726 n = 543455 S(n-2) = 1693 S(n-1) = 967 S(n) = 241

same difference = 306 n = 565187 S(n-2) = 919 S(n-1) = 613 S(n) = 307
same difference = 552 n = 574498 S(n-2) = 1381 S(n-1) = 829 S(n) = 277
same difference = 1176 n = 586272 S(n-2) = 2549 S(n-1) = 1373 S(n) = 197
same difference = 3444 n = 592537 S(n-2) = 6971 S(n-1) = 3527 S(n) = 83
same difference = 390 n = 609871 S(n-2) = 1091 S(n-1) = 701 S(n) = 311
same difference = 840 n = 629508 S(n-2) = 1931 S(n-1) = 1091 S(n) = 251
same difference = 336 n = 681077 S(n-2) = 1009 S(n-1) = 673 S(n) = 337
same difference = 612 n = 705793 S(n-2) = 1531 S(n-1) = 919 S(n) = 307
same difference = 78 n = 724319 S(n-2) = 467 S(n-1) = 389 S(n) = 311
same difference = 264 n = 726827 S(n-2) = 881 S(n-1) = 617 S(n) = 353
same difference = 498 n = 731179 S(n-2) = 1327 S(n-1) = 829 S(n) = 331
same difference = 240 n = 751746 S(n-2) = 839 S(n-1) = 599 S(n) = 359
same difference = 1356 n = 778837 S(n-2) = 2939 S(n-1) = 1583 S(n) = 227
same difference = 1020 n = 792675 S(n-2) = 2311 S(n-1) = 1291 S(n) = 271
same difference = 2214 n = 803427 S(n-2) = 4591 S(n-1) = 2377 S(n) = 163
same difference = 1590 n = 810451 S(n-2) = 3391 S(n-1) = 1801 S(n) = 211
same difference = 252 n = 837969 S(n-2) = 883 S(n-1) = 631 S(n) = 379
same difference = 552 n = 898783 S(n-2) = 1471 S(n-1) = 919 S(n) = 367
same difference = 2850 n = 930311 S(n-2) = 5851 S(n-1) = 3001 S(n) = 151
same difference = 540 n = 941057 S(n-2) = 1459 S(n-1) = 919 S(n) = 379
same difference = 240 n = 943553 S(n-2) = 881 S(n-1) = 641 S(n) = 401
same difference = 120 n = 975546 S(n-2) = 619 S(n-1) = 499 S(n) = 379
same difference = 1122 n = 997443 S(n-2) = 2551 S(n-1) = 1429 S(n) = 307
same difference = 264 n = 1026550 S(n-2) = 947 S(n-1) = 683 S(n) = 419
same difference = 684 n = 1028985 S(n-2) = 1747 S(n-1) = 1063 S(n) = 379
same difference = 744 n = 1042162 S(n-2) = 1861 S(n-1) = 1117 S(n) = 373
same difference = 510 n = 1053175 S(n-2) = 1429 S(n-1) = 919 S(n) = 409